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BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

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It is of interest to inquire whether gauge vector mesons acquire mass through interaction[1]; by a gauge vector meson we mean a Yang-Mills field[2] associated with the extension of a Lie group from global to local symmetry. The importance of this problem resides in the possibility that strong-interaction physics originates from massive gauge fields related to a system of conserved currents.[3] In this note, we shall show that in certain cases vector mesons do indeed acquire mass when the vacuum is degenerate with respect to a compact Lie group.

Theories with degenerate vacuum (broken symmetry) have been the subject of intensive study since their inception by Nambu.[4-6] A characteristic feature of such theories is the possible existence of zero-mass bosons which tend to restore the symmetry.[7,8] We shall show that it is precisely these singularities which maintain the gauge invariance of the theory, despite the fact that the vector meson acquires mass.

We shall first treat the case where the original fields are a set of bosons φ_A which transform as a basis for a representation of a compact Lie group. This example should be considered as a rather general phenomenological model. As such, we shall not study the particular mechanism by which the symmetry is broken but simply assume that such a mechanism exists. A calculation performed in lowest order perturbation theory indicates that those vector mesons which are coupled to currents that "rotate" the original vacuum are the ones which acquire mass [see Eq. (6)].

We shall then examine a particular model based on chirality invariance which may have a more fundamental significance. Here we begin with a chirality-invariant Lagrangian and introduce both vector and pseudovector gauge fields, thereby guaranteeing invariance under both local phase and local γ_5-phase transformations. In this model the gauge fields themselves may break the γ_5 invariance leading to a mass for the original Fermi field. We shall show in this case that the pseudovector field acquires mass.

In the last paragraph we sketch a simple argument which renders these results reasonable.

(1) Lest the simplicity of the argument be shrouded in a cloud of indices, we first consider a one-parameter Abelian group, representing, for example, the phase transformation of a charged boson; we then present the generalization to an arbitrary compact Lie group.

The interaction between the φ and the A_μ fields is

$$H_{\text{int}} = ieA_\mu \varphi^* \overleftrightarrow{\partial}_\mu \varphi - e^2 \varphi^* \varphi A_\mu A_\mu, \tag{1}$$

where $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$. We shall break the symmetry by fixing $\langle\varphi\rangle \neq 0$ in the vacuum, with the phase chosen for convenience such that $\langle\varphi\rangle = \langle\varphi^*\rangle = \langle\varphi_1\rangle/\sqrt{2}$.

We shall assume that the application of the theorem of Goldstone, Salam, and Weinberg[7] is straightforward and thus that the propagator of the field φ_2, which is "orthogonal" to φ_1, has a pole at $q=0$ which is not isolated.

We calculate the vacuum polarization loop $\Pi_{\mu\nu}$ for the field A_μ in lowest order perturbation theory about the self-consistent vacuum. We take into consideration only the broken-symmetry diagrams (Fig. 1). The conventional terms do not lead to a mass in this approximation if gauge invariance is carefully maintained. One evaluates directly

$$\Pi_{\mu\nu}(q) = (2\pi)^4 ie^2[g_{\mu\nu}\langle\varphi_1\rangle^2 - (q_\mu q_\nu/q^2)\langle\varphi_1\rangle^2]. \quad (2)$$

Here we have used for the propagator of φ_2 the value $[i/(2\pi)^4]/q^2$; the fact that the renormalization constant is 1 is consistent with our approximation.[9] We then note that Eq. (2) both maintains gauge invariance ($\Pi_{\mu\nu}q_\nu=0$) and causes the A_μ field to acquire a mass

$$\mu^2 = e^2\langle\varphi_1\rangle^2. \quad (3)$$

We have not yet constructed a proof in arbitrary order; however, the similar appearance of higher order graphs leads one to surmise the general truth of the theorem.

Consider now, in general, a set of boson-field operators φ_A (which we may always choose to be Hermitian) and the associated Yang-Mills field $A_{a,\mu}$. The Lagrangian is invariant under the transformation[10]

$$\delta\varphi_A = \sum_{a,A}\epsilon_a(x)T_{a,AB}\varphi_B,$$

$$\delta A_{a,\mu} = \sum_{c,b}\epsilon_c(x)c_{acb}A_{b,\mu} + \partial_\mu\epsilon_a(x), \quad (4)$$

where c_{abc} are the structure constants of a compact Lie group and $T_{a,AB}$ the antisymmetric generators of the group in the representation defined by the φ_B.

Suppose that in the vacuum $\langle\varphi_{B'}\rangle \neq 0$ for some B'. Then the propagator of $\sum_{A,B'}T_{a,AB'}\varphi_A$ $\times\langle\varphi_{B'}\rangle$ is, in the lowest order,

$$\left[\frac{i}{(2\pi)^4}\right]\sum_{A,B',C'}\frac{T_{a,AB'}\langle\varphi_{B'}\rangle T_{a,AC'}\langle\varphi_{C'}\rangle}{q^2}$$

$$\equiv\left[\frac{-i}{(2\pi)^4}\right]\frac{(\langle\varphi\rangle T_a T_a\langle\varphi\rangle)}{q^2}.$$

With λ the coupling constant of the Yang-Mills field, the same calculation as before yields

$$\Pi_{\mu\nu}{}^a(q) = -i(2\pi)^4\lambda^2(\langle\varphi\rangle T_a T_a\langle\varphi\rangle)$$

$$\times[g_{\mu\nu} - q_\mu q_\nu/q^2],$$

giving a value for the mass

$$\mu_a{}^2 = -(\langle\varphi\rangle T_a T_a\langle\varphi\rangle). \quad (6)$$

(2) Consider the interaction Hamiltonian

$$H_{\rm int} = -\eta\bar{\psi}\gamma_\mu\gamma_5\psi B_\mu - \epsilon\bar{\psi}\gamma_\mu\psi A_\mu, \quad (7)$$

where A_μ and B_μ are vector and pseudovector gauge fields. The vector field causes attraction whereas the pseudovector leads to repulsion between particle and antiparticle. For a suitable choice of ϵ and η there exists, as in Johnson's model,[11] a broken-symmetry solution corresponding to an arbitrary mass m for the ψ field fixing the scale of the problem. Thus the fermion propagator $S(p)$ is

$$S^{-1}(p) = \gamma p - \Sigma(p) = \gamma p[1-\Sigma_2(p^2)] - \Sigma_1(p^2), \quad (8)$$

with

$$\Sigma_1(p^2) \neq 0$$

and

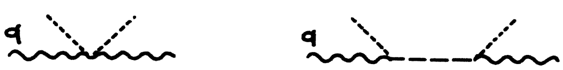
$$m[1-\Sigma_2(m^2)] - \Sigma_1(m^2) = 0.$$

We define the gauage-invariant current $J_\mu{}^5$ by using Johnson's method[12]:

$$J_\mu{}^5 = -\eta\lim_{\xi\to 0}\bar{\psi}'(x+\xi)\gamma_\mu\gamma_5\psi'(x),$$

$$\psi'(x) = \exp[-i\int_{-\infty}^x \eta B_\mu(y)dy^\mu\gamma_5]\psi(x). \quad (9)$$

This gives for the polarization tensor of the



(a) (b)

FIG. 1. Broken-symmetry diagram leading to a mass for the gauge field. Short-dashed line, $\langle\varphi_1\rangle$; long-dashed line, φ_2 propagator; wavy line, A_μ propagator. (a) $\to (2\pi)^4 ie^2 g_{\mu\nu}\langle\varphi_1\rangle^2$, (b) $\to -(2\pi)^4 ie^2(q_\mu q_\nu/q^2)\times\langle\varphi_1\rangle^2$.

pseudovector field

$$\Pi_{\mu\nu}{}^5(q) = \eta^2 \frac{i}{(2\pi)^4} \int \mathrm{Tr}\{S(p-\tfrac{1}{2}q)\Gamma_{\nu 5}(p-\tfrac{1}{2}q; p+\tfrac{1}{2}q) \times S(p+\tfrac{1}{2}q)\gamma_\mu \gamma_5 - S(p)[\partial S^{-1}(p)/\partial p_\nu] S(p)\gamma_\mu\} d^4p, \quad (10)$$

where the vertex function $\Gamma_{\nu 5} = \gamma_\nu \gamma_5 + \Lambda_{\nu 5}$ satisfies the Ward identity[5]

$$q_\nu \Lambda_{\nu 5}(p-\tfrac{1}{2}q; p+\tfrac{1}{2}q) = \Sigma(p-\tfrac{1}{2}q)\gamma_5 + \gamma_5 \Sigma(p+\tfrac{1}{2}q), \quad (11)$$

which for low q reads

$$q_\nu \Gamma_{\nu 5} = q_\nu \gamma_\nu \gamma_5 [1-\Sigma_2] + 2\Sigma_1 \gamma_5 - 2(q_\nu p_\nu)(\gamma_\lambda p_\lambda)(\partial \Sigma_2/\partial p^2)\gamma_5. \quad (12)$$

The singularity in the longitudinal $\Gamma_{\nu 5}$ vertex due to the broken-symmetry term $2\Sigma_1\gamma_5$ in the Ward identity leads to a nonvanishing gauge-invariant $\Pi_{\mu\nu}{}^5(q)$ in the limit $q \to 0$, while the usual spurious "photon mass" drops because of the second term in (10). The mass of the pseudovector field is roughly $\eta^2 m^2$ as can be checked by inserting into (10) the lowest approximation for $\Gamma_{\nu 5}$ consistant with the Ward identity.

Thus, in this case the general feature of the phenomenological boson system survives. We would like to emphasize that here the symmetry is broken through the gauge fields themselves. One might hope that such a feature is quite general and is possibly instrumental in the realization of Sakurai's program.[3]

(3) We present below a simple argument which indicates why the gauge vector field need not have zero mass in the presence of broken symmetry. Let us recall that these fields were introduced in the first place in order to extend the symmetry group to transformations which were different at various space-time points. Thus one expects that when the group transformations become homogeneous in space-time, that is $q \to 0$, no dynamical manifestation of these fields should appear. This means that it should cost no energy to create a Yang-Mills quantum at $q = 0$ and thus the mass is zero. However, if we break gauge invariance of the first kind and still maintain gauge invariance of the second kind this reasoning is obviously incorrect. Indeed, in Fig. 1, one sees that the A_μ propagator connects to intermediate states, which are "rotated" vacua. This is seen most clearly by writing $\langle\varphi_1\rangle = \langle[Q\varphi_2]\rangle$ where Q is the group generator. This effect cannot vanish in the limit $q \to 0$.

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